Opener

Calculator

If the derivative of f is given by $f'(x) = e^x - 3x^2$, at which of the following values of x does f have a relative maximum value?

(A) -0.46 (B) 0.20 (C) 0.91 (D) 0.95 (E) 3.73

Non-Calculator

What are all values of x for which the function f defined by $f(x) = (x^2 - 3)e^{-x}$ is increasing?

(A) There are no such values of x.

(B) $x < -1$ and $x > 3$

(C) $-3 < x < 1$

(D) $-1 < x < 3$

(E) All values of x

4-3 day 2 The Concavity Test

Learning Objecves:

I can idenfy an inflecon on a graph and I understand the relaonship between this point and the derivaves of the funcon.

I can use the concavity test to find points of inflecon.

I can idenfy the intervals on which a funcon is concave up or concave down.

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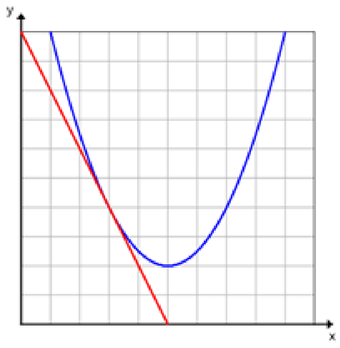
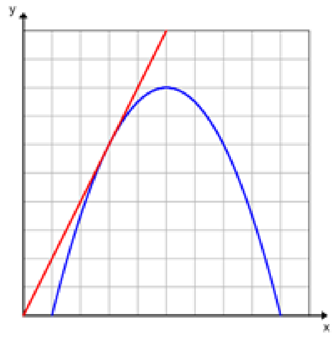


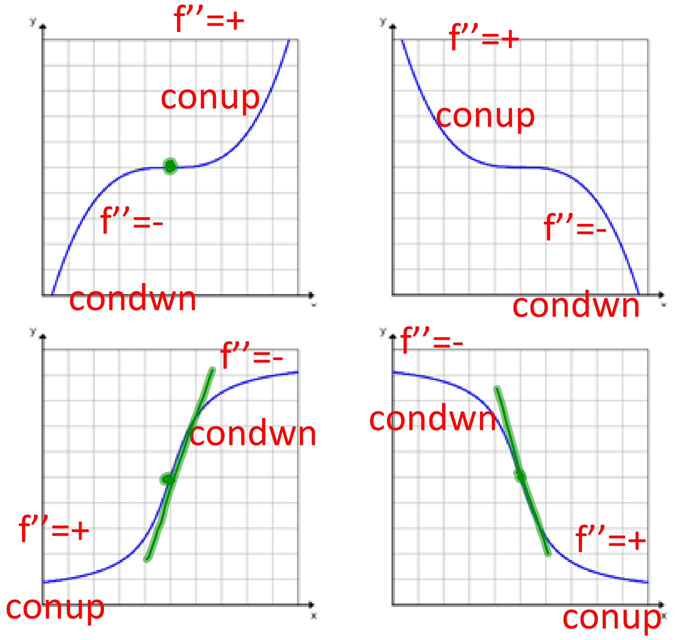


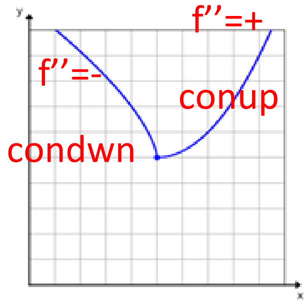
concave upwards

$f'' = +$

concave downwards

$f'' = -$





This is not an inflecon point because even though the concavity changes, no tangent line exists.

An ***inflecon point*** is an point on the graph of a funcon where the concavity changes AND a tangent line exists.

Concavity Test

$f''(x)>0$ f(x) is concave up

$f''(x)<0$ f(x) is concave down

2nd derivaves find concavity (tell us if the funcon is concave up or concave down) and are used to find inflecon points.

A sign change in the second derivave indicates a change in concavity. You must observe a sign change to be sure that an inflecon point is present.

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Ex1. Use the concavity test to determine the intervals on which the graph is concave up or concave down. Idenfy any inflecon points.

1.) $f(x) = x^3 - 4x^2 - 3x - 5$

$f' = 3x^2 - 8x - 3$

$f'' = 6x - 8$

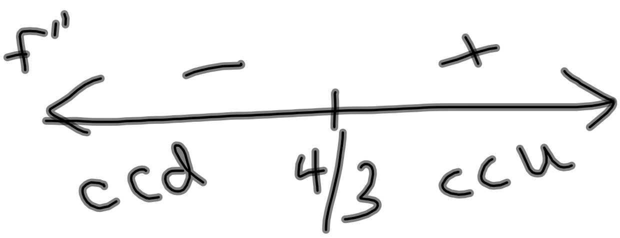
$0 = 6x - 8$

$x = \frac{4}{3}$

candidates

$f'' = 0$: $x = 4/3$

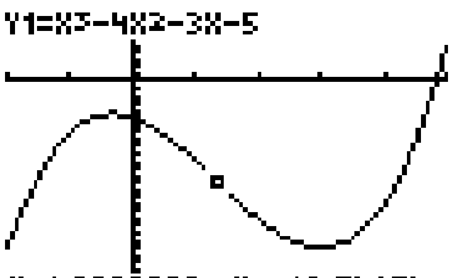
f'' = undef:



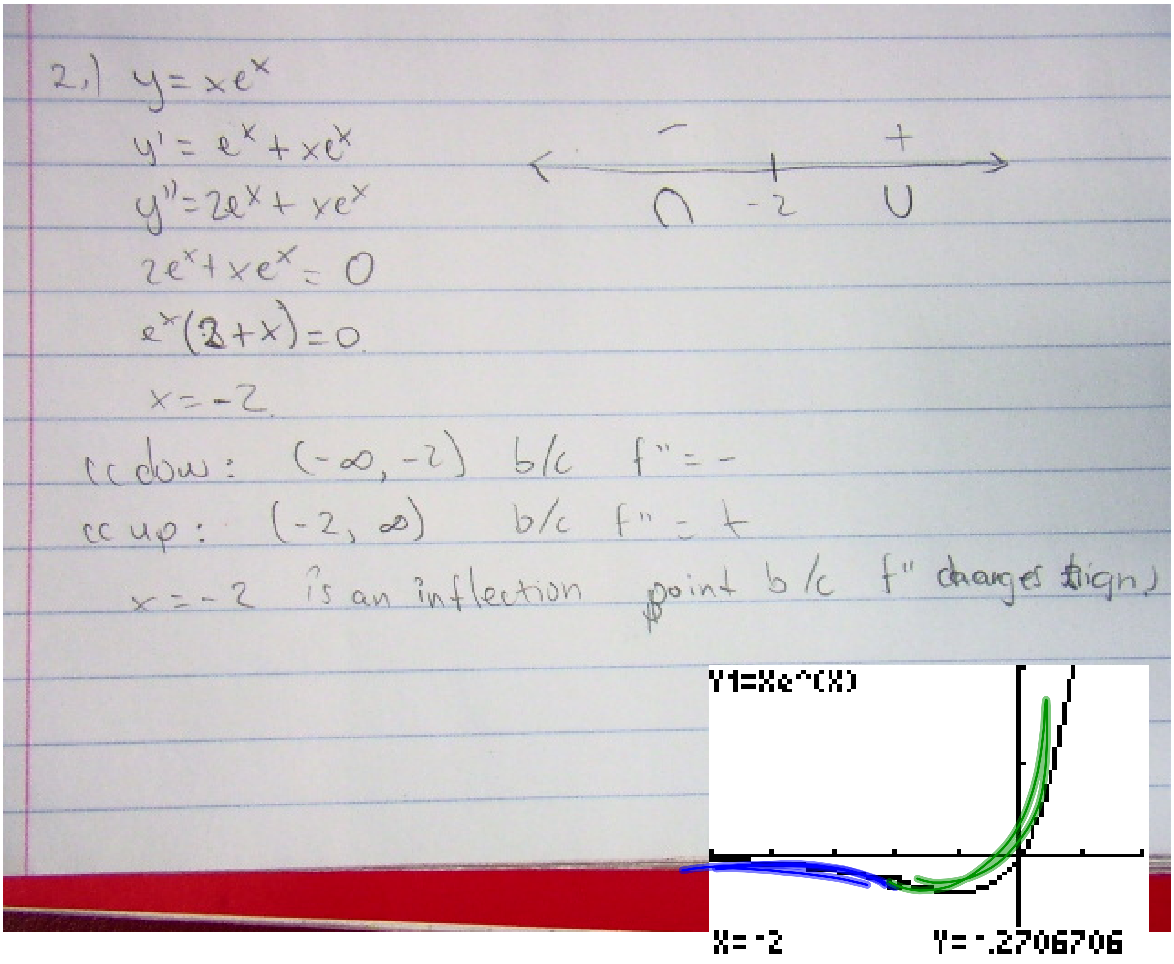
cc down: $(-\infty, 4/3)$ b/c $f'' = -$

cc up: $(4/3, \infty)$ b/c $f'' = +$

$x = \frac{4}{3}$ is an inf. pt b/c f'' changes signs



2.) $y = xe^x$


2.) $y = xe^x$

$y' = e^x + xe^x$

$y'' = 2e^x + xe^x$

$2e^x + xe^x = 0$

$e^x(2+x) = 0$

$x = -2$

cc down: $(-\infty, -2)$ b/c $f'' = -$

cc up: $(-2, \infty)$ b/c $f'' = +$

$x = -2$ is an inflection point b/c f'' changes sign.

D: $[-3,\infty)$

3.) $y = x\sqrt{x+3}$

$y = x(x+3)^{1/2}$ (f = x, g = $(x+3)^{1/2}$)

$y' = 1(x+3)^{1/2} + x \cdot \frac{1}{2}(x+3)^{-1/2}$

$y' = (x+3)^{1/2} + \frac{1}{2}x \cdot (x+3)^{-1/2}$ (f = $\frac{1}{2}x$, g = $(x+3)^{-1/2}$)

$y'' = \frac{1}{2}(x+3)^{-1/2} + \frac{1}{2}(x+3)^{-1/2} + \frac{1}{2}x \cdot -\frac{1}{2}(x+3)^{-3/2}$

$y'' = \frac{1}{2}(x+3)^{-1/2} + \frac{1}{2}(x+3)^{-1/2} - \frac{1}{4}x(x+3)^{-3/2}$

$y'' = (x+3)^{-1/2} - \frac{1}{4}x(x+3)^{-3/2}$

$y'' = \frac{1}{\sqrt{x+3}} - \frac{x}{4\sqrt{(x+3)^3}}$ $\qquad \frac{1}{\sqrt{3}} - \frac{0}{\#}$

$\left[0 = \frac{1}{\sqrt{(x+3)^1}} - \frac{x}{4\sqrt{(x+3)^3}} \right] 4\sqrt{(x+3)^3}$

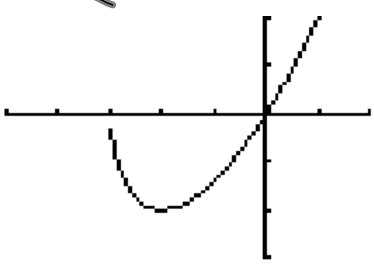
$0 = 4\sqrt{(x+3)^2} - x$

$0 = 4(x+3) - x$

$0 = 4x + 12 - x$

$0 = 3x + 12$ $\qquad$ ccu $(-3,\infty)$

~~$x = -4$~~



Homework

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