

# Opener

## Calculator

If the derivative of  $f$  is given by  $f'(x) = e^x - 3x^2$ , at which of the following values of  $x$  does  $f$  have a relative maximum value?

- (A)  $-0.46$       (B)  $0.20$       (C)  $0.91$       (D)  $0.95$       (E)  $3.73$

## Non-Calculator

What are all values of  $x$  for which the function  $f$  defined by  $f(x) = (x^2 - 3)e^{-x}$  is increasing?

- (A) There are no such values of  $x$  .  
(B)  $x < -1$  and  $x > 3$   
(C)  $-3 < x < 1$   
(D)  $-1 < x < 3$   
(E) All values of  $x$

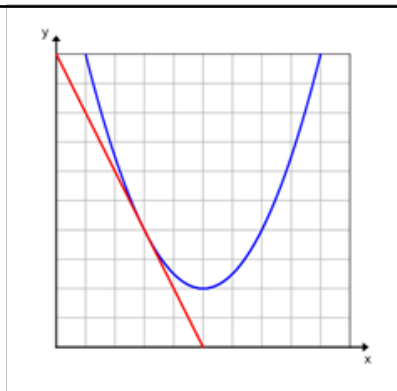
# 4-3 day 2 The Concavity Test

## Learning Objectives:

I can identify an inflection on a graph and I understand the relationship between this point and the derivatives of the function.

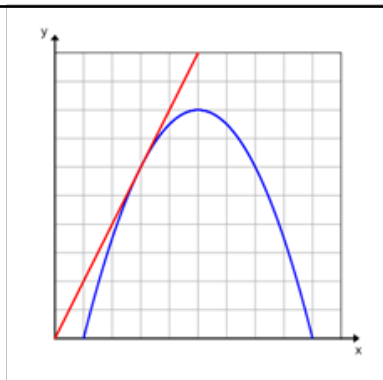
I can use the concavity test to find points of inflection.

I can identify the intervals on which a function is concave up or concave down.



**concave upwards**

$$f'' = +$$



**concave downwards**

$$f'' = -$$

The image contains four coordinate planes, each with a blue curve and red text labels:

- Top-left graph:** A curve that is concave down ( $f'' = -$ ) on the left and concave up ( $f'' = +$ ) on the right. A green dot marks the point where the concavity changes, labeled as an inflection point. Labels include "conup", "conown", "f''=-", and "f''=+".
- Top-right graph:** A curve that is concave up ( $f'' = +$ ) on the left and concave down ( $f'' = -$ ) on the right. The point where the concavity changes is not marked. Labels include "conup", "conown", "f''=+", and "f''=-".
- Bottom-left graph:** A curve that is concave up ( $f'' = +$ ) on the left and concave down ( $f'' = -$ ) on the right. A green tangent line is drawn at the point where the concavity changes, labeled as an inflection point. Labels include "conup", "conown", "f''=+", and "f''=-".
- Bottom-right graph:** A curve that is concave down ( $f'' = -$ ) on the left and concave up ( $f'' = +$ ) on the right. A green tangent line is drawn at the point where the concavity changes, labeled as an inflection point. Labels include "conown", "conup", "f''=-", and "f''=+".

This is not an inflection point because even though the concavity changes, no tangent line exists.

An ***inflection point*** is a point on the graph of a function where the concavity changes AND a tangent line exists.

## Concavity Test

$f''(x) > 0$   $f(x)$  is concave up

$f''(x) < 0$   $f(x)$  is concave down

2<sup>nd</sup> derivatives find concavity (tell us if the function is concave up or concave down) and are used to find inflection points.

A sign change in the second derivative indicates a change in concavity. You must observe a sign change to be sure that an inflection point is present.

Ex1. Use the concavity test to determine the intervals on which the graph is concave up or concave down. Identify any inflection points.

$$1.) f(x) = x^3 - 4x^2 - 3x - 5$$

$$f' = 3x^2 - 8x - 3$$

$$f'' = 6x - 8$$

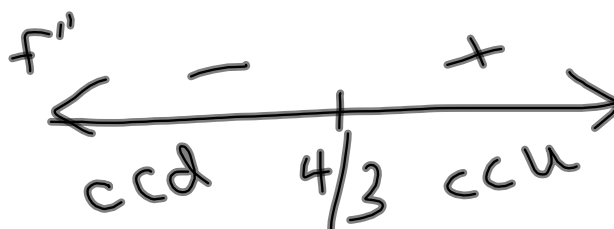
$$0 = 6x - 8$$

$$x = \frac{4}{3}$$

candidates

$$f'' = 0: x = \frac{4}{3}$$

$$f'' = \text{undef.}$$

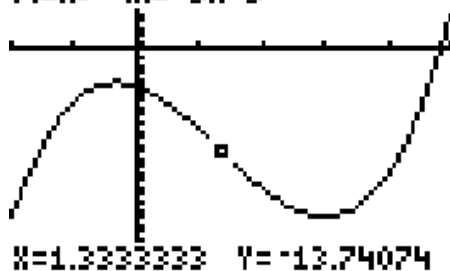


cc down:  $(-\infty, \frac{4}{3})$  b/c  $f'' = -$

cc up:  $(\frac{4}{3}, \infty)$  b/c  $f'' = +$

$x = \frac{4}{3}$  is an inf. pt  
b/c  $f''$  changes signs

$$Y1 = X^3 - 4X^2 - 3X - 5$$



$$2.) y = x e^x$$

$$2.) y = x e^x$$

$$y' = e^x + x e^x$$

$$y'' = 2e^x + x e^x$$

$$2e^x + x e^x = 0$$

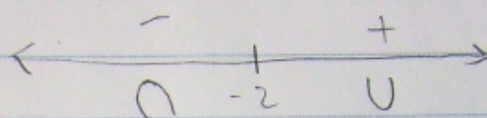
$$e^x(2+x) = 0$$

$$x = -2$$

cc down:  $(-\infty, -2)$  b/c  $f'' = -$

cc up:  $(-2, \infty)$  b/c  $f'' = +$

$x = -2$  is an inflection point b/c  $f''$  changes sign.



$Y1 = X e^X$

$X = -2$

$Y = -.2706706$

$$3.) y = x\sqrt{x+3} \quad D: [-3, \infty)$$

$$y = \underbrace{x}_f \cdot \underbrace{(x+3)^{1/2}}_g$$

$$y' = 1 \cdot (x+3)^{1/2} + x \cdot \frac{1}{2} (x+3)^{-1/2}$$

$$y' = (x+3)^{1/2} + \frac{1}{2}x \cdot (x+3)^{-1/2}$$

$$y'' = \frac{1}{2}(x+3)^{-1/2} + \frac{1}{2} \cdot (x+3)^{-1/2} + \frac{1}{2}x \cdot \frac{-1}{2}(x+3)^{-3/2}$$

$$y'' = \frac{1}{2}(x+3)^{-1/2} + \frac{1}{2}(x+3)^{-1/2} - \frac{1}{4}x(x+3)^{-3/2}$$

$$y'' = (x+3)^{-1/2} - \frac{1}{4}x(x+3)^{-3/2}$$

$$y'' = \frac{1}{\sqrt{x+3}} - \frac{x}{4\sqrt{(x+3)^3}} \quad \frac{1}{\sqrt{3}} - \frac{0}{\#}$$

$$\left[ 0 = \frac{1}{\sqrt{x+3}} - \frac{x}{4\sqrt{(x+3)^3}} \right] \quad 4\sqrt{(x+3)^3}$$

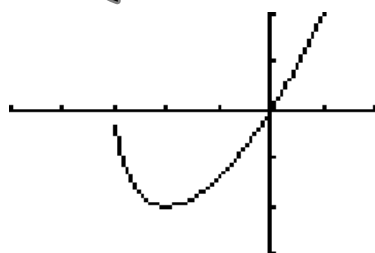
$$0 = 4\sqrt{(x+3)^2} - x$$

$$0 = 4(x+3) - x$$

$$0 = 4x + 12 - x$$

$$0 = 3x + 12 \quad \text{CCU } (-3, \infty)$$

~~$$x = -4$$~~





# Homework

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